

Number Systems

Exercises

1. $1010 + 1101$ [Answer: 10111](#)
2. $1011 + 111$ [Answer: 10010](#)
3. $1111 - 1010$ [Answer: 101](#)
4. $1110 - 101$ [Answer: 1001](#)
5. 1101×101 [Answer: 1000001](#)
6. $1001 \div 101$ [Answer: 1r101 or 1.11001100... which we write 1. \$\dot{1}\$ 10 \$\dot{0}\$](#)
7. What is the biggest binary number you can write with 5 bits? [Answer: 31](#)
8. What is the biggest binary number you can write with n bits? [Answer: \$2^n - 1\$](#)
9. Roughly, how many bits do you need to write the number n in binary? [Answer: roughly \$\log_2 n\$, exactly \$\lceil \log_2\(n+1\) \rceil\$](#)
10. Write $\frac{3}{4}$ in binary, using a “binary point” 0.???. [Answer: 0.11](#)
11. Write $\frac{2}{3}$ in binary. [Answer: \$0.101010\dots = 0.1\dot{0}\$](#)
12. Which fractions recur infinitely in binary and which terminate? [Answer: if demoninator of fraction in lowest form is power of 2 then it terminates, else not.](#)

Exercises

13. Convert the binary number 1101101111110101 to hex. [Answer: DBF5](#)
14. Convert the hex number ABC7 to binary. [Answer: 101010110111](#)
15. In hex, $2BFC + 54A7$. [Answer: 80A3](#)
16. In hex, $AC74 - B3F$. [Answer: A135](#)
17. If a number has k digits in hex, how many digits (bits) does it have in binary? [Answer: \$\frac{k}{4}\$](#)

18. If a number has k digits in decimal, roughly how many digits does it have in binary? **Answer:** roughly $\frac{k}{\log_2 10} \cong \frac{k}{3}$
19. If a number has k digits base a , roughly how many digits does it have in base b ? **Answer:** roughly $\frac{k}{\log_b a} = k \times \log_a b$

Exercises Write the following decimal numbers in eight bit two's complement, do the addition/subtraction, convert your answer back to decimal.

20. $-3 + +11$ **Answer:** $-3 = 11111101$. So $-3 + +11 = 11111101 + 00001011 = 00001000$
21. $+125 + -40$ **Answer:** $125 = 01111101$, $-40 = 11011000$ so $125 + -40 = 01010101 = 85$
22. $+5 + -7$ **Answer:** $00000101 + 11111001 = 11111110 = -2$
23. $-23 + -18$. **Answer:** $11101001 + 11101110 = 11010111 = -41$

For n bit two's complement what is the range of numbers you can represent?

Answer: Biggest positive number is $0 \overbrace{11 \dots 1}^{n-1} = 2^{n-1} - 1$. Negative numbers go down from $-1 = \overbrace{1 \dots 11}^n$ down to $1 \overbrace{00 \dots 01}^{n-2} = -2^{n-1} - 1$. But negative numbers can go one lower still: least negative number is $1 \overbrace{00 \dots 0}^{n-1} = -2^{n-1}$. So range of numbers is $[-2^{n-1}, +2^{n-1} - 1]$.

Exercises Write the following fractions in binary. Use one sign bit and four bits after the binary point.

24. $\frac{5}{8}$ **Answer:** 0.1010
25. $-\frac{5}{8}$ **Answer:** negate previous answer, i.e. flip all bits and add one to lsb, so 1.0110

Convert the following binary fractions to ordinary fractions.

26. 0.1000 **Answer:** $\frac{1}{2}$
27. 1.0001 **Answer:** $-\frac{15}{16}$
28. 0.1111 **Answer:** $\frac{15}{16}$
29. 1.1111 **Answer:** $-\frac{1}{16}$

What fractions can be represented as binary fractions with one sign bit and four bits after the binary point? **Answer:** $[-1, +\frac{15}{16}]$

Exercises Using 5 bits for the mantissa and 5 bits for the exponent, write the following numbers in twos complement binary.

30. $\frac{5}{16}$ Answer: 0.0101 0000, mantissa represents $\frac{5}{16}$ exponent represents $2^0 = 1$
31. $101\frac{1}{4}$ Answer: Bad example. This number is $\frac{21}{4}$, so mantissa should be $\frac{21}{32} = 0.10101$, but four available bits are not enough to represent this. Can approximate by losing last bit, 0.1010 0011, i.e. $\frac{5}{8} \times 2^3 = 5$, so the quarter gets lost.
32. $\frac{1}{1024}$ Answer: 0.1000 1011
33. $-\frac{3}{512}$ Answer: 0.0011 11001